Please answer questions 1 - 4 refer to the following recurrence relation.

$B(1) = 3$
$B(n) = 2B(n-1)$ for all $n > 1$

1. Please write the first five terms in the sequence

B(1) = 3
B(2) = 2B(1) = 6
B(3) = 2B(2) = 12
B(4) = 2B(3) = 24
B(5) = 2B(4) = 48

2. Write the python cody of a recursive function to solve the relation above

In [3]:
```
def B(n):
    if n == 1:
        return 3
    else:
        return 2*B(n-1)
```

In [6]:
```
for i in range (1,6):
    print("B(",i,") = ", B(i))
```

```
B( 1 ) =  3
B( 2 ) =  6
B( 3 ) =  12
B( 4 ) =  24
B( 5 ) =  48
```

3. Write a for loop to solve the relation above

In [4]:
```
def Bloop(n):
        if n == 1:
            return 3;
        else:
            B = 3
            for i in range(2,n+1):
                B = 2*B;
            return B
```

In [7]:
```
for i in range (1,6):
    print("B(",i,") = ", Bloop(i))
```

```
B( 1 ) =  3
B( 2 ) =  6
B( 3 ) =  12
B( 4 ) =  24
B( 5 ) =  48
```

4. Please find the closed form solution using the linear, first-order recurrence relation with constant coefficients formula:

$$S(n) = c^{n-1}S(1) + \sum_{i=2}^{n} c^{n-i}g(i)$$

B(1) = 3
B(n) = 2*B(n-1)

S(n) = cS(n-1) + g(n)
c = 2
g(n) = 0

$S(n) = 2^{n-1} * 3 + 0 = 2^{n-1} * 3$

Please answer questions 5 - 6 refer to the following recurrence relation.

$S(1) = 3$
$S(n) = S(n-1) + n$ for all $n > 1$

5. Using the formula in Q4, find the closed-form formula for the given recurrence relation.

S(1) = 3
c = 1
g(n) = n

$1^{n-1} * 3 + \sum_{i=2}^{n} 1^{n-i} * i$
$3 + \sum_{i=2}^{n} i$

6. Simplify the formula in Q5 using summation facts

$3 + (2 + 3 + \ldots + n)$
$3 - 1 + (1 + 2 + 3 + \ldots + n)$

$2 + \frac{n(n+1)}{2}$

Summation Facts

(1) $\sum_{i=m}^{n} c = (n - m + 1)c$

(2) $\sum_{i=m}^{n} ca_i = c \sum_{i=m}^{n} a_i$

(3) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

(4) $\sum_{i=1}^{n} i^2 = \frac{n(n+1))(2n+1)}{6}$

In [ ]: