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### CSCI-3080 Discrete Structure
### OLA 2: Chapter 2 -- Proofs, Induction
### Name:
### Student ID:
### Date:
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Please use Proofs, Induction to finish the following 6 exercises. (100 points)

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Exercise 1: Provide a counter example to the following statement: (4 points)

The number n is an odd integer if and only if $3n + 5$ is an even integer.

This is actually the statement: (If n is an odd integer then $3n + 5$ is an even integer) AND (If $3n + 5$ is an even integer, then n is an odd integer)

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Exercise 2: Please prove the following statement using exhaustive proof. (12 points)

For 2 ≤ n ≤ 4 , $n^2 \geq 2^n$

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Exercise 3: Please prove the following statement using direct proof (12 points)

The sum of an even integer and an odd integer is odd.

Rewrite mathematically: If x is even and y is odd, then $x + y$ is odd.

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Exercise 4: Please prove the following statement using proof by contradiction. (12 points)

If $x^2 + 2x - 3 = 0$, then x ≠ 2

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Exercise 5: For all positive integers, let $P(n)$ be the equation: (20 points)

$$2 + 4 + 6 + \cdots + 2n = n(n+1)$$

a. Write the equation for the base case $P(1)$ and verify that it is true.
b. Write the inductive hypothesis $P(k)$
c. Write the equation for $P(k+1)$
d. Prove that $P(k+1)$ is true given b

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Exercise 6: Please prove the following statements are true for every positive integer n using mathematical induction. (40 points)

(1) $2 + 6 + 18 + \cdots + 2 \times 3^{n-1} = 3^n - 1$ (20 points)

(2) $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \cdots + n \times 2^n = (n-1) \times 2^{n+1} + 2$ (20 points)

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